

November 14, 2014

# Determination of $|V_{us}|$ from the $\tau$ lepton branching fractions

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We determine the Cabibbo-Kobayashi-Maskawa matrix (CKM) element  $|V_{us}|$  in several different ways using updated preliminary HFAG averages for the  $\tau$  lepton branching fractions and we compare the results with the determinations obtained from the kaon decays and from the unitarity of the CKM matrix.

PRESENTED AT

8th International Workshop on the CKM Unitarity Triangle  
(CKM 2014), Vienna, Austria, September 8–12, 2014

# 1 Introduction

The CKM matrix element  $|V_{us}|$  is most precisely determined from kaon decays [1], and its precision is limited by the uncertainties of the lattice QCD estimates of  $f_+(0)$  and  $f_K/f_\pi$ . Using the  $\tau$  branching fractions it is possible to determine  $|V_{us}|$  in an alternative way [2] that does not depend on lattice QCD and has small theory uncertainties:

$$|V_{us}| = \sqrt{R_s / \left[ \frac{R_{\text{VA}}}{|V_{ud}|^2} - \delta R_{\text{theory}} \right]}, \quad (1)$$

where  $R_s$  and  $R_{\text{VA}}$  are the ratio of the inclusive  $\tau$  width into strange and non-strange final states to the partial width into electron ( $\Gamma(\tau \rightarrow e\nu_\tau\bar{\nu}_e)$ ). In this formula, the  $SU(3)$  breaking parameter  $\delta R_{\text{theory}}$  depends on the  $s$  quark mass and is determined in the context of low energy QCD theory, partly relying on experimental data on low energy QCD processes [2]. The required experimental data, including uncertainties and correlations, are adequately provided by the HFAG averages [3]. In the following, we update the HFAG averages to September 2014 and we determine  $|V_{us}|$  accordingly. Furthermore, we use the updated HFAG data to determine  $|V_{us}|$  with  $\tau$  decays in two additional ways, using  $\text{BR}(\tau \rightarrow K\nu)$ . This provides an alternative determination of  $|V_{us}|$  that is independent of the kaon data but is affected in a similar way by the parameters  $f_K$  and  $f_\pi$  (determined with lattice QCD).

## 2 $|V_{us}|$ from inclusive $\tau \rightarrow X_s\nu$

Referring to Eq. 1, we use preliminary HFAG averages updated to September 2014 to compute the experimental inputs. Following Ref. [4], we assume lepton universality to obtain a the “universality-improved”  $\text{BR}_e = \text{BR}(\tau \rightarrow e\bar{\nu}_e\nu_\tau)$  using the  $\tau$  branching fraction to electron and muon and the  $\tau$  lifetime, including the precise 2013 Belle result [5], obtaining  $\text{BR}_e^{\text{uni}} = (17.815 \pm 0.023)\%$ . We get  $R_s = \text{BR}(\tau \rightarrow X_s\nu)/\text{BR}_e^{\text{uni}} = 0.1618 \pm 0.0026$  (see also Table 1) and  $R_{\text{VA}} = \text{BR}(\tau \rightarrow X_{\text{non-strange}}\nu)/\text{BR}_e^{\text{uni}} = 3.4696 \pm 0.0080$ . We update  $\delta R_{\text{theory}}$  from Ref. [2] with the up-to-date value of the  $s$ -quark mass  $m_s = 93.50 \pm 2.50$  [6] and get  $\delta R_{\text{theory}} = 0.239 \pm 0.030$ . We chose not to use more recent calculations of  $\delta R_{\text{theory}}$  [7, 8], which have smaller or larger estimated uncertainties. Using  $|V_{ud}| = 0.97425 \pm 0.00022$  [6], we compute  $|V_{us}|_{\tau s} = 0.2176 \pm 0.0021$ , which is  $3.4\sigma$  lower than the unitarity CKM prediction  $|V_{us}|_{\text{uni}} = 0.22547 \pm 0.00095$ , from  $(|V_{us}|_{\text{uni}})^2 = 1 - |V_{ud}|^2$  ( $|V_{ub}|$  being negligible). The uncertainty contribution from  $\delta R_{\text{theory}}$  is 0.44 %.

Branching fraction	HFAG preliminary Summer 2014 fit
$\Gamma_{10} = K^- \nu_\tau$	$(0.6955 \pm 0.0096) \cdot 10^{-2}$
$\Gamma_{16} = K^- \pi^0 \nu_\tau$	$(0.4331 \pm 0.0149) \cdot 10^{-2}$
$\Gamma_{23} = K^- 2\pi^0 \nu_\tau$ (ex. $K^0$ )	$(0.0630 \pm 0.0220) \cdot 10^{-2}$
$\Gamma_{28} = K^- 3\pi^0 \nu_\tau$ (ex. $K^0, \eta$ )	$(0.0419 \pm 0.0216) \cdot 10^{-2}$
$\Gamma_{35} = \pi^- \bar{K}^0 \nu_\tau$	$(0.8378 \pm 0.0123) \cdot 10^{-2}$
$\Gamma_{40} = \pi^- \bar{K}^0 \pi^0 \nu_\tau$	$(0.3680 \pm 0.0103) \cdot 10^{-2}$
$\Gamma_{44} = \pi^- \bar{K}^0 \pi^0 \pi^0 \nu_\tau$ (ex. $K^0$ )	$(0.0124 \pm 0.0204) \cdot 10^{-2}$
$\Gamma_{53} = \bar{K}^0 h^- h^- h^+ \nu_\tau$	$(0.0222 \pm 0.0202) \cdot 10^{-2}$
$\Gamma_{128} = K^- \eta \nu_\tau$	$(0.0153 \pm 0.0008) \cdot 10^{-2}$
$\Gamma_{130} = K^- \pi^0 \eta \nu_\tau$	$(0.0048 \pm 0.0012) \cdot 10^{-2}$
$\Gamma_{132} = \pi^- \bar{K}^0 \eta \nu_\tau$	$(0.0093 \pm 0.0015) \cdot 10^{-2}$
$\Gamma_{151} = K^- \omega \nu_\tau$	$(0.0410 \pm 0.0092) \cdot 10^{-2}$
$\Gamma_{801} = K^- \phi \nu_\tau (\phi \rightarrow KK)$	$(0.0037 \pm 0.0014) \cdot 10^{-2}$
$\Gamma_{802} = K^- \pi^- \pi^+ \nu_\tau$ (ex. $K^0, \omega$ )	$(0.2923 \pm 0.0068) \cdot 10^{-2}$
$\Gamma_{803} = K^- \pi^- \pi^+ \pi^0 \nu_\tau$ (ex. $K^0, \omega, \eta$ )	$(0.0411 \pm 0.0143) \cdot 10^{-2}$
$\Gamma_{822} = K^- 2\pi^- 2\pi^+ \nu_\tau$ (ex. $K^0$ )	$(0.0001 \pm 0.0001) \cdot 10^{-2}$
$\Gamma_{833} = K^- 2\pi^- 2\pi^+ \pi^0 \nu_\tau$ (ex. $K^0$ )	$(0.0001 \pm 0.0001) \cdot 10^{-2}$
$\Gamma_{110} = X_s^- \nu_\tau$	$(2.8816 \pm 0.0470) \cdot 10^{-2}$

Table 1: HFAG preliminary Summer 2014  $\tau$  branching fractions to strange final states.

## 2.1 $|V_{us}|$ from $\text{BR}(\tau \rightarrow K\pi\nu)$

It is also possible to determine  $|V_{us}|$  from the branching fraction  $\text{BR}(\tau \rightarrow K\pi\nu)$  [9, 10] from the equation:

$$\Gamma(\tau \rightarrow \bar{K}\pi\nu_\tau[\gamma]) = \frac{G_F^2 m_\tau^5}{96\pi^3} C_K^2 S_{\text{EW}}^\tau (|V_{us}| f_+^{K\pi}(0))^2 \times \\ \times I_K^\tau \left(1 + \delta_{\text{EM}}^{K\tau} + \tilde{\delta}_{\text{SU}(2)}^{K\pi}\right)^2.$$

The phase space integrals,  $I_K^\ell$  are determined from the  $K\pi$  form factors. The first estimate of the long-distance electromagnetic corrections ( $\delta_{\text{EM}}^{K\tau}$ ) have been computed [9]. The isospin breaking corrections ( $\tilde{\delta}_{\text{SU}(2)}^{K\pi}$ ) allow using both the measurement on  $\tau \rightarrow K^- \pi^0 \nu_\tau$  and  $\tau \rightarrow K_S^0 \pi \nu_\tau$ . Using data from the HFAG 2012 fit, from kaon decays,  $f_+^{K\pi}(0)$  from FLAG 2013 [11], one obtains  $f_+^{K\pi}(0)|V_{us}| = 0.2141 \pm 0.0021_{\text{exp}} \pm 0.0014_{IK\tau}$

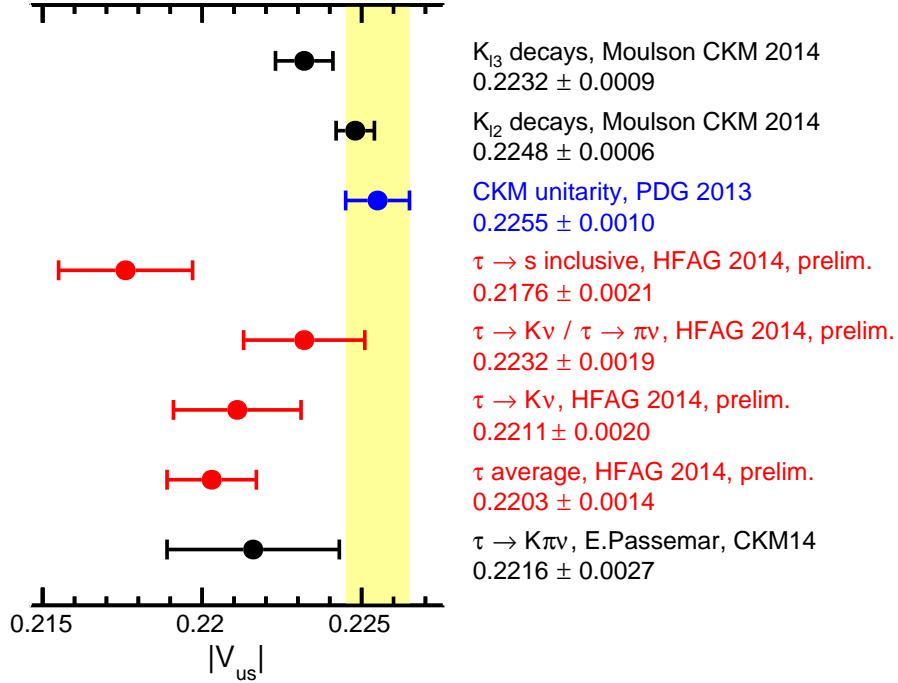


Figure 1:  $|V_{us}|$  from tau and kaon decays and from the CKM matrix unitarity.

and  $|V_{us}| = 0.2216 \pm 0.0027$  [10]. Neglecting correlations, this is  $1.4\sigma$  below the CKM unitarity prediction.

### 3 Conclusions

All determinations of  $|V_{us}|$  from  $\tau$  decays are lower than the CKM matrix unitarity determination. In particular,  $|V_{us}|$  from inclusive  $\tau \rightarrow X_s \nu$  is  $3.4\sigma$  lower. Averaging the results using the HFAG 2014 preliminary data,  $|V_{us}|_{\tau s}$ ,  $|V_{us}|_{\tau K/\pi}$  and  $|V_{us}|_{\tau K}$ , we obtain:

$$|V_{us}|_{\tau} = 0.2204 \pm 0.0014 ,$$

$2.9\sigma$  lower than the CKM unitarity prediction. The correlation of the uncertainties on  $f_K$  and  $f_K/f_\pi$  is neglected, since we could not find any estimate if it. Even assuming  $\pm 100\%$  correlation, the uncertainty on  $|V_{us}|_{\tau}$  does not change more than about  $\pm 5\%$ .

The  $|V_{us}|$  values obtained from kaon decays [1], on the other hand, are statistically consistent with the unitarity determination. Matthew Moulson has presented in the same conference updated and more precise values of  $|V_{us}|$  from kaon decays [12], which

continue to be compatible with the unitarity determination. Figure 1 summarizes the  $|V_{us}|$  results, using the most up-to-date kaon results.

## References

- [1] M. Antonelli *et al.*, Eur. Phys. J. **C69**, 399 (2010), arXiv:1005.2323 [hep-ph].
- [2] E. Gamiz, M. Jamin, A. Pich, J. Prades, and F. Schwab, Nucl. Phys. Proc. Suppl. **169**, 85 (2007), arXiv:hep-ph/0612154.
- [3] Y. Amhis *et al.*, (2012), arXiv:1207.1158 [hep-ex].
- [4] M. Davier, A. Hocker, and Z. Zhang, Rev. Mod. Phys. **78**, 1043 (2006), arXiv:hep-ph/0507078 [hep-ph].
- [5] Belle, K. Belous *et al.*, Phys. Rev. Lett. **112**, 031801 (2014), arXiv:1310.8503 [hep-ex].
- [6] J. Beringer *et al.*, Phys. Rev. **D86**, 010001 (2012), and 2013 partial update for the 2014 edition.
- [7] E. Gamiz, M. Jamin, A. Pich, J. Prades, and F. Schwab, PoS **KAON**, 008 (2008), arXiv:0709.0282 [hep-ph].
- [8] K. Maltman, Nucl. Phys. Proc. Suppl. **218**, 146 (2011), arXiv:1011.6391 [hep-ph].
- [9] M. Antonelli, V. Cirigliano, A. Lusiani, and E. Passemar, JHEP **1310**, 070 (2013), arXiv:1304.8134 [hep-ph].
- [10] E. Passemar, Determinations of  $|V_{us}|$  from hadronic tau decays: A theory perspective, presented at the 8th International Workshop on the CKM Unitarity Triangle (CKM 2014), Vienna, Austria, September 8–12, 2014, to be published in the CKM 2014 conference proceedings.
- [11] S. Aoki *et al.*, Eur. Phys. J. **C74**, 2890 (2014), arXiv:1310.8555 [hep-lat].
- [12] M. Moulson, Experimental determination of  $|V_{us}|$  from kaon decays, presented at the 8th International Workshop on the CKM Unitarity Triangle (CKM 2014), Vienna, Austria, September 8–12, 2014, to be published in the CKM 2014 conference proceedings.